Imbalance Compensation for an AMB System with Adaptive Immersion & Invariance Control

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Abstract

This study is concerned with the compensation of imbalance force existing in a three-pole active magnetic bearing (AMB) system. An adaptive compensation scheme is proposed using the method of immersion and invariance (I&I) control. The main idea of I&I control is to immerse a higher-order system into a lower-order target system. In this study, the target system will be the three-pole AMB system with known imbalance that is stabilized by an integral sliding mode controller (ISMC). In practical applications, the imbalance force may be unknown and may be changing during operation. The information of imbalance needs to be estimated online in order to realize the ISMC levitation control. To this aim, the adaptive compensation scheme is incorporated into the ISMC controller so that the closed-loop system forms the higher-order system for the design of I&I control. Numerical simulations verify the effectiveness of the proposed adaptive compensator for mass imbalance.

1 Introduction

Mass imbalance is an important issue for rotating machinery. The induced imbalance force will increase in a square way as the rotation speed increases. For contact bearings, the imbalance force will be transferred to the stator and base through the bearing, causing structural vibrations. Active magnetic bearing (AMB) is an excellent device for resolving the problem of mass imbalance. The imbalance force can be actively compensated and hence it will not be transferred to the structure.

There have been many studies on the imbalance compensation of AMB systems. The most popular approach is adaptive feedforward compensation. This approach regards the imbalance force as an external disturbance that needs to be compensated. There are many methods with this approach, such as observer-based method [10], notch filer [3], least square optimization [8], and sliding mode control [12]. Another approach is to view the mass imbalance as a property of the system. It can be estimated using the technique of system identification. Then, the controller of the AMB can be tuned according to the estimated mass imbalance [4, 9]. The above approaches are to control the rotor's geometric center to the bearing center. This is the requirement for most precision rotating machineries. To this aim, we need to provide additional force (and energy) to compensate fro the imbalance force. A different approach is to control the rotor to spin about its principal axis of inertia [11]. As such, the imbalance force will automatically vanish. No additional compensation force is required which can save energy. This approach is useful for flywheel energy storage system [13].

The objective of this study is to propose an imbalance compensator for a three-pole a AMB system [2, 5, 6]. Since the three-pole AMB system is a strongly nonlinear system, an integral sliding mode controller (ISMC) is first designed for stable levitation, with the assumption of known imbalance. In practical applications, the imbalance force may be unknown and may be changing during operation. The information of imbalance needs to be estimated online in order to realize the levitation control. To this aim, the method of adaptive immersion and invariance (I&I) control is adopted here [1]. The main idea of I&I control is to immerse a higher-order system (the AMB system with the imbalance adaptation law) into a lower-order one (the one with known mass imbalance and with only ISMC). It is desired to design an appropriate adaptive I&I controller to cancel the imbalance force in the three-pole AMB system.

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2 System Description and Robust Stabilization

Figure 1 is the 3-pole AMB system considered in this study. It is a 2-DOF system with a disk-like rigid rotor. The axial motion is constrained with thrust ball bearings. The AMB is Y-shaped with differential windings on the upper two poles to yield the optimal design from the viewpoint of energy and cost [2]. It is assumed that the magnetic field is linear, flux leakage and fringing effects are negligible, and gravitational field *g* is in the negative *y* direction. The system has been investigated previously in the literature. For more details, please refer to [2, 5, 6]. In this study, it is assumed that imbalance mass exists in the system. As a result, the mass center of the rotor does not coincide with its geometric center, as shown in Figure 2. The system dynamics can be obtained with these assumptions, as

$$\dot{x} = \begin{bmatrix} x_2 \\ c_0\Phi_1\Phi_2 \\ x_4 \\ \frac{c_0}{2}[\Phi_2^2 - \Phi_1^2] - g \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \Omega^2\cos\Omega t & -\Omega^2\sin\Omega t \\ 0 & 0 \\ \Omega^2\sin\Omega t & \Omega^2\cos\Omega t \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}$$

where $x = [x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T = [x_r \;\; \dot{x}_r \;\; y_r \;\; \dot{y}_r]^T$ is the state vector containing the rotor displacements x_r, y_r and their velocities, (a, b) is the position of mass center relative to the geometric center at $t = 0$, and Ω is the constant rotor speed. The coefficient c_0 is defined by

$$c_0 = \frac{4\mu A N^2}{3m}$$

where μ is the magnetic permeability of the air, A is pole face area, and N is the number of coil turns for each pole and m is the rotor mass. The functions Φ_1 and Φ_2 are quantities related to magnetic flux. They depend on the rotor displacements and coil currents in the following way

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \frac{-1}{L} \begin{bmatrix} 2l_0 - x_3 & \sqrt{3}x_1 \\ x_1 & \sqrt{3}(2l_0 + x_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{2}$$

where l_0 is the nominal air gap and $L = 4l_0^2 - (x_1^2 + x_3^2)$ is always positive in the operation range because that the rotor displacement is always smaller than the nominal air gap, i.e. $(x_1^2 + x_3^2) \le l_0^2$. The determinant of the matrix in (2) is $\sqrt{3}L$ which is always nonzero. Thus the coil currents i_1, i_2 can be expressed in terms of Φ_1, Φ_2 as

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{-1}{\sqrt{3}} \begin{bmatrix} \sqrt{3}(2l_0 + x_3) & -\sqrt{3}x_1 \\ -x_1 & (2l_0 - x_3) \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} \tag{3}$$

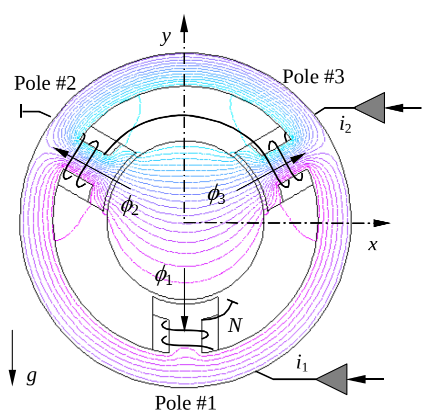

Figure 1: The 3-pole AMB system

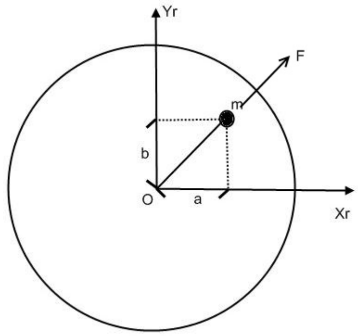

Figure 2: Illustration of the imbalance mass

Without the imbalance mass (i.e., $a = b = 0$), the robust stabilization of the present system (1) has been studied previously in [], using the feedback linearization technique and integral sliding mode control (ISMC). In this section, we will first assume that the position of the imbalanced mass center (a, b) is known. Then, the compensation of the imbalance mass can be easily achieved by the ISMC. As a result, the robust stabilization controller with the imbalance compensation is given by

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{-1}{\sqrt{3c_0}} \begin{bmatrix} \sqrt{3}(2l_0 + x_3) & -\sqrt{3}x_1 \\ -x_1 & 2l_0 - x_3 \end{bmatrix} \begin{bmatrix} \sqrt{-(\tilde{i}_2 + g) + \sqrt{(\tilde{i}_2 + g)^2 + \tilde{i}_1^2}}\ \mathrm{sgn}(\tilde{i}_1) \\ \sqrt{(\tilde{i}_2 + g) + \sqrt{(\tilde{i}_2 + g)^2 + \tilde{i}_1^2}} \end{bmatrix} \tag{4}$$

where $\mathrm{sgn}(\bullet)$ is the sign function and

$$\begin{bmatrix} \tilde{i}_1 & \tilde{i}_2 \end{bmatrix}^T = \tilde{i} = -b_1\xi - b_2\eta - \frac{\rho + \alpha}{1 - k} \mathrm{sat}\left(\frac{\sigma}{\varepsilon}\right) - \Omega^2 R(t)\theta_* \tag{5}$$

where $\eta = [\eta_1\ \eta_2]^T = [x_1\ x_3]^T$, $\xi = [\xi_1\ \xi_2]^T = [x_2\ x_4]^T$, α and ε are positive constants, and ρ and $0 \le k < 1$ are the uncertainty bounds, and the integral sliding manifold is defined by

$$\sigma = \xi + b_1\eta + b_2\zeta; \quad \dot{\zeta} = \eta \tag{6}$$

where b_1 and b_2 are positive constants. The vector type saturation function is defined by

$$sat\left(\frac{\sigma}{\varepsilon}\right) = \left[sat\left(\frac{\sigma_1}{\varepsilon}\right)\ \ sat\left(\frac{\sigma_2}{\varepsilon}\right) \right]^T$$

The last term in Equation (5) is for the compensation of the imbalance mass, where

$$R(t) = \begin{bmatrix} \cos\Omega t & -\sin\Omega t \\ \sin\Omega t & \cos\Omega t \end{bmatrix}; \ \theta_* = \begin{bmatrix} a & b \end{bmatrix}^T \tag{7}$$

Equation (4) is used to feedback linearize the nonlinear magnetic forces in Equation (1). On the other hand, Equation (5) is the ISMC that can yield the robust stabilization.

3 Adaptive Imbalance Compensation

In many practical applications, the position of the imbalanced mass center (a, b) is unknown, or cannot be precisely known. In this case, the compensation provided in Equation (5) will not be precise. The mismatched imbalance force will cause steady state errors. As the rotation speed increases, the imbalance force will also increase and can damage the system. To deal with the unknown (a, b), an adaptive compensator is designed in this section. The design will be based on the theory of immersion and invariance [1], which will be briefly introduced below.

Consider an affine nonlinear system given by

$$\dot{x} = f(x,t) + g(x)u \tag{8}$$

where $x \in \Re^n$ is the state vector and $u \in \Re^m$ is the input vector. Suppose that the system (8) can be linearly parameterized in a set of unknown parameters $\theta_* \in \Re^p$ as

$$f(x,t) = f_0(x,t) + f_1(x,t)\theta_* \tag{9}$$

Suppose also that there exists a control law

$$u = \Psi(x, t, \theta_*) \tag{10}$$

such that the closed-loop system

$$\dot{x} = f(x,t) + g(x)\Psi(x, t, \theta_*) \tag{11}$$

possesses an asymptotically stable equilibrium at the origin. Then, the theory of immersion and invariance is to seek a function $\beta_1(x, t)$ such that

(i) the dynamics

$$\dot{z} = -\left[\frac{\partial \beta_1(x,t)}{\partial x} f_1(x,t)\right] z \tag{12}$$

will result in bounded responses for $z(t)$.

(ii) $\lim_{t\to\infty} g(x(t))[\Psi(x(t),t,z(t)+\theta_*) - \Psi(x(t),t,\theta_*)] = 0$ (13)

With the function $\beta_1(x,t)$, we can generate another function $\beta_2(x, t, \hat{\theta})$ by

$$\beta_2(x, t, \hat{\theta}) = -\frac{\partial \beta_1(x,t)}{\partial x}\left(f_0(x, t) + f_1(x, t)\left[\hat{\theta} + \beta_1(x,t)\right] + g(x)\Psi(x,t,\hat{\theta})\right) \tag{14}$$

With the two functions $\beta_1(x,t)$ and $\beta_2(x, t, \hat{\theta})$ in hand, an adaptive control law modified from the control law (10) can be constructed as

$$u = \Psi(x, t, \hat{\theta} + \beta_1(x, t)); \quad \dot{\hat{\theta}} = \beta_2(x, t, \hat{\theta}) \tag{15}$$

Next, we will apply the above method to the present system. The feedback linearized system of the original system (1) by (4) is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ 0 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \Omega^2 \cos\Omega t & -\Omega^2 \sin\Omega t \\ 0 & 0 \\ \Omega^2 \sin\Omega t & \Omega^2 \cos\Omega t \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{i}_1 \\ 0 \\ \tilde{i}_2 \end{bmatrix} \tag{16}$$

which is in the form of (8) and (9) with

$$f_0(x) = \begin{bmatrix} 0 & x_2 & 0 & x_4 \end{bmatrix}^T; \; f_1(x,t) = \Omega^2 \begin{bmatrix} 0 & \cos\Omega t & 0 & \sin\Omega t \\ 0 & -\sin\Omega t & 0 & \cos\Omega t \end{bmatrix}^T; \; g(x) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$\theta_* = \begin{bmatrix} a & b \end{bmatrix}^T; \; u = \tilde{i} = \begin{bmatrix} \tilde{i}_1 & \tilde{i}_2 \end{bmatrix}^T$$

On the other hand, the stabilization control is given by (5) and (6), i.e.,

$$\Psi(x, t, \theta_*) = -b_1\xi - b_2\eta - \frac{\rho + \alpha}{1-k}\text{sat}\left(\frac{\sigma}{\varepsilon}\right) - \Omega^2 R(t)\theta_*; \; \sigma = \xi + b_1\eta + b_2\zeta; \quad \dot{\zeta} = \eta \tag{17}$$

To satisfy the condition (i), we take

$$\frac{\partial \beta_1(x,t)}{\partial x} = (h/\Omega^2) f_1^T(x,t) \tag{18}$$

where $h > 0$ is a tunable gain. That is,

$$\beta_1(x,t) = h\begin{bmatrix} x_2 \cos\Omega t + x_4 \sin\Omega t \\ -x_2 \sin\Omega t + x_4 \cos\Omega t \end{bmatrix} \tag{19}$$

With (18), Equation (12) becomes

$$\dot{z} = -h\Omega^2 z \tag{20}$$

It is obvious that the response of $z(t)$ is bounded and

$$\lim_{t\to\infty} z(t) = 0 \tag{21}$$

Hence, condition (i) is satisfied. Moreover,

$$g(x(t))[\Psi(x(t),t,z(t)+\theta_*)-\Psi(x(t),t,\theta_*)]=-\Omega^2\begin{bmatrix}0&1&0&0\\0&0&0&1\end{bmatrix}^T R(t)z \quad (22)$$

Therefore, by (21), we have

$$\lim_{t\to\infty} g(x(t))[\Psi(x(t),t,z(t)+\theta_*)-\Psi(x(t),t,\theta_*)]=0 \quad (23)$$

implying that condition (ii) is also satisfied. Using the function given by Equation (19), one can get

$$\beta_2\left(x,t,\hat{\theta}\right)=-\frac{\partial\beta_1(x,t)}{\partial x}\left(f_0(x,t)+f_1(x,t)\left[\hat{\theta}+\beta_1(x,t)\right]+g(x)\Psi\left(x,t,\hat{\theta}\right)\right)=hR^T(t)\left[\left(b_1-h\Omega^2\right)\xi+b_2\eta+\frac{\rho+\alpha}{1-k}\text{sat}\left(\frac{\sigma}{\varepsilon}\right)\right] \quad (24)$$

Therefore, the stabilizing adaptive control law can be obtained as

$$\tilde{i}=-\left(b_1+h\Omega^2\right)\xi-b_2\eta-\frac{\rho+\alpha}{1-k}\text{sat}\left(\frac{\sigma}{\varepsilon}\right)-\Omega^2R(t)\hat{\theta}\,;\quad \dot{\hat{\theta}}=\beta_2\left(x,t,\hat{\theta}\right)=hR^T(t)\left[\left(b_1-h\Omega^2\right)\xi+b_2\eta+\frac{\rho+\alpha}{1-k}\text{sat}\left(\frac{\sigma}{\varepsilon}\right)\right] \quad (25)$$

4 Simulation Results

The system parameters for numerical simulations are given by

m=0.634 kg, g=9.81 m/s^2, l_0=0.95×10^{-3} m, μ=4π×10^{-7} H/m, A=4×10^{-4} m^2, N=300

Assume that the position of the imbalanced mass center is

$$\theta_*=\begin{bmatrix}a & b\end{bmatrix}^T=\begin{bmatrix}1\times10^{-5}(m) & 2\times10^{-5}(m)\end{bmatrix}^T$$

The control gains for the ISMC levitation controller are

$$b_1=25\quad b_2=250\quad \varepsilon=0.25\quad \alpha=4\quad k=0.5\quad \rho=4.51$$

and the gain for the adaptive I&I control is taken to be $h=1\times10^{-4}$. The initial condition for the system states is

$$x(0)=\begin{bmatrix}0 & 0 & -5.5\times10^{-4}(m) & 0\end{bmatrix}^T$$

which assumes that the rotor is initially at rest on the back up bearing. The initial guess for the imbalance mass center is set to be

$$\hat{\theta}(0)=\begin{bmatrix}5\times10^{-6}(m) & 5\times10^{-6}(m)\end{bmatrix}^T$$

In what follows, three stages of simulations are performed. First, at $t=0\,\text{sec}$, the ISMC controller is activated to levitate the rotor to the bearing center. Next, at $t=2\,\text{sec}$, the motor is activated to rotate the rotor at the given speed of $\Omega\ rad/s$. Finally, at $t=4\,\text{sec}$, the adaptive I&I control is activated to compensate for the imbalance mass. Three cases of rotation speeds are considered, i.e., $\Omega=500\ rad/s, 1000\ rad/s, 2000\ rad/s$. Figure 3 shows the rotor displacement response for the case with the rotation speed of 1000 rad/s, and Figure 4 is the corresponding rotor trajectory. In Figure 4, the dashed line represents the region constrained by the back up bearing. As one can see from the figures, at the first stage, the ISMC controller can levitate the rotor and stabilize it within 0.3 sec. When the motor is activated, the rotor will start to vibrate, roughly in the order of imbalanced mass center (around $20\mu m$). Then, such steady state vibrations can be eliminated within 1.5 sec when the adaptive I&I control is introduced. The simulation results for all three cases are summarized in Table 1. The results show that the steady state root of mean square (rms) error can be significantly reduced by the adaptive I&I control. For the case of 2000 rad/s, the rms error is reduced from 22 μm to 1.8 μm. For lower rotation speeds, the error can even be reduced to 0.1 μm. These results demonstrate the effectiveness of the proposed imbalance compensator.

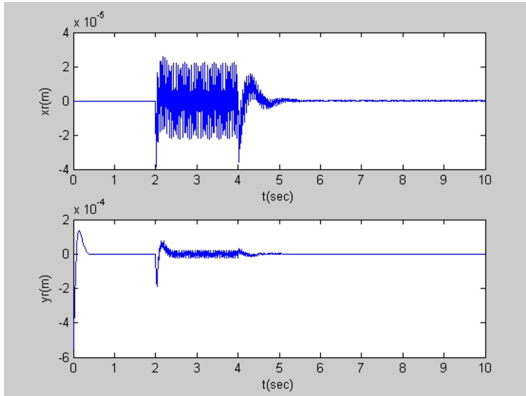

Figure 3: Rotor displacement response with $\Omega = 1000\ rad/s$

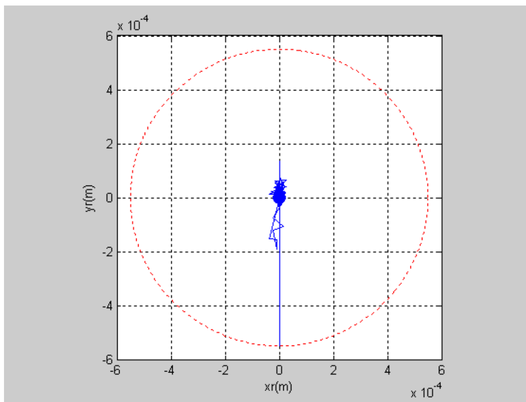

Figure 4: Rotor trajectory with $\Omega = 1000\ rad/s$

Rotation speed	Steady state rms error without compensation	Steady state rms error with compensation
500rad/s	$2.22\times10^{-5}(m)$	$1.06\times10^{-7}(m)$
1000rad/s	$2.23\times10^{-5}(m)$	$1.00\times10^{-7}(m)$
2000rad/s	$2.24\times10^{-5}(m)$	$1.80\times10^{-6}(m)$

Table 1: Steady state errors with and without imbalance compensation

5 Conclusions

An adaptive compensator for mass imbalance existing in a three-pole AMB system has been proposed in this study. The adaptive compensation scheme is designed based on the method of I&I control. The three-pole AMB system with an ISMC levitation controller and the adaptive compensation scheme is considered as a higher-order system. The method of I&I control is applied to design the adaptive compensation scheme so that this higher-order system can be immersed into a lower-order target system. The target system will be the three-pole AMB system with known imbalance that is stabilized by the ISMC controller. Numerical simulations are carried out for three cases of rotation speeds. It is found that without compensation, the steady state rotor vibration is about 22 μm. With the proposed adaptive compensation, it can be significantly reduced to 1.8 μm for case of 2000 rad/s, and 0.1 μm for both cases of 500 rad/s and 1000 rad/s. These results clearly verify the effectiveness of the proposed adaptive compensator for mass imbalance.

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