

# Duality parameter of Approximate Dual Control for Electromagnetic Suspension

Kento Osako\* and Tetsuzo Sakamoto\*\*

\*Graduate School of Engineering,

\*\* Faculty of Engineering

Graduate School, Kyushu Institute of Technology

Tobata, Kitakyushu, Fukuoka 804-8550, Japan

## Abstract

In dual control, the control input has a dual role so that the system with uncertain parameters should be controlled carefully while obtaining quality system information to identify. This paper deals with the effects of our approximate method of dual control on the performance of electromagnetic suspension system. This method compensates a weakness of model predictive control which requires an accuracy system model. The performance weight between the control and the identification is formulated by a single design parameter. We show simulations when varying the design parameter of magnitude of excitation in order to study the effects on the control and identification result.

**Keywords** : electromagnetic suspension, dual control, duality parameter, self-sensing

## 1. Introduction

Noncontact electromagnetic suspension (EMS) technique has been used in various fields such as levitated vehicle, satellite and artificial heart. The system is unstable, and therefore feedback controller is needed for stable levitation. To achieve stable levitation, it is necessary to obtain high quality system model. In this paper, we study the effects of the duality parameter for the tradeoff between control and identification on the EMS performance, in which the approximate dual control establishes a simultaneous optimization for control and identification (Matsuda and Sakamoto, 2015). The information matrix is used to obtain quality system information for the identification. The duality parameter is formulated as the weight between control and identification. The performance simulations are shown by varying duality parameter in order to change the level of excitation.

## 2. Approximate dual control formulation

The dual controller includes three blocks: (a) Kalman filter to estimate the system parameters, (b) Model predictive controller, and (c) Excitation unit for quality identification.

### 2.1 Model predictive control

We use auto regressive exogenous (ARX) model for the dual control as follows:

$$y(k) = \sum_{j=1}^{n_b} b_j u(k - n_b) - \sum_{i=1}^{n_a} a_i y(k - n_a) = \boldsymbol{\theta}^T \mathbf{Z}(k) \quad (1)$$

$$\boldsymbol{\theta} = (b_1 \quad \cdots \quad b_j \quad -a_1 \quad \cdots \quad -a_i)^T \quad (2)$$

$$\mathbf{Z}(k) = (u(k-1) \quad \cdots \quad u(k-n_b) \quad y(k-1) \quad \cdots \quad y(k-n_a))^T \quad (3)$$

where  $\mathbf{Z}(k)$  is the regression vector containing the sequences of input  $u(k)$  and output  $y(k)$ ,  $\boldsymbol{\theta}$  the parameter vector,  $n_a$  and  $n_b$  denote the number of lagged inputs and outputs respectively. The parameter vector  $\boldsymbol{\theta}$  is estimated by using the Kalman filter, where the filtering algorithm is given by the following recursive equations:

$$\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \mathbf{K}(k)\{y(k+1) - \hat{\boldsymbol{\theta}}^T(k)\mathbf{Z}(k)\} \quad (4)$$

$$\mathbf{K}(k+1) = \mathbf{P}(k)\mathbf{Z}(k)[\mathbf{Z}^T(k)\mathbf{P}(k)\mathbf{Z}(k) + \sigma_\xi^2]^{-1} \quad (5)$$

$$\mathbf{P}(k+1) = \mathbf{P}(k) - \mathbf{K}(k+1)\mathbf{Z}^T(k)\mathbf{P}(k) + \mathbf{Q} \quad (6)$$

where  $\hat{\boldsymbol{\theta}}(k)$  is the estimated parameter vector,  $\mathbf{K}(k)$  the Kalman gain,  $\mathbf{P}(k)$  the covariance matrix of  $\boldsymbol{\theta}(k)$ ,  $\sigma_\xi^2$  the variance of white noise, and  $\mathbf{Q}$  the covariance matrix of system noise.

Then, the model predictive control (MPC) calculates the control input sequence using the estimated parameter vector. The MPC problem is formulated as follows:

$$\mathbf{U}_c(k) = \arg \min_{\mathbf{U}_c(k)} J_c(\mathbf{U}_c(k)) \quad (7)$$

$$J_c(\mathbf{U}_c(k)) = \sum_{k=0}^{H_c} \|Q_{MPC}\{y(k) - r\}\|_2^2 + \|R_{MPC}u_{ck}\|_2^2 \quad (8)$$

subject to

$$y(k) = \hat{\boldsymbol{\theta}}^T(k)\mathbf{Z}(k) \quad (9)$$

$$\mathbf{U}_c(k) = (u_{c1} \ u_{c2} \ \cdots \ u_{cH_c})^T, u_{min} \leq u_{ck} \leq u_{max}, k = 1, \dots, H_c \quad (10)$$

where  $\mathbf{U}_c(k)$  is the control input vector,  $Q_{MPC}$  and  $R_{MPC}$  the positive weighting matrices,  $r$  the reference trajectory,  $u_{min}$  and  $u_{max}$  the hard constraint on the system input. The MPC is effective with the ideal model, however, the numerical conditions of the system parameter can vary every second or is uncertain due to sensor noise. Therefore the MPC needs quality information to estimate the system parameters.

## 2.2 Maximization of information matrix

In order to search for quality identification, we employ maximization of the minimum eigenvalue of the information matrix (Rathousky and Havlena, 2006) (Zacekova et al, 2013). The algorithm is given as follows.

$$\mathbf{U}(k) = \arg \max_{\mathbf{U}(k)} J(\mathbf{U}(k)) \quad (11)$$

$$J(\mathbf{U}(k)) = \lambda_{\min} \left( \sum_{k=0}^{H_c} \mathbf{Z}(k)\mathbf{Z}^T(k) \right) \quad (12)$$

subject to

$$y(k) = \hat{\boldsymbol{\theta}}^T(k)\mathbf{Z}(k) \quad (13)$$

$$\mathbf{U}(k) = (u_1 \ u_{c2} \ u_3 \ \cdots \ u_{H_c})^T, u_{min} \leq u_k \leq u_{max}, k = 1, \dots, H_c \quad (14)$$

$$\sum_{k=0}^{H_c} \|Q_{MPC}\{y(k) - r\}\|_2^2 + \|R_{MPC}u_k\|_2^2 \leq \alpha J_c(\mathbf{U}_c(k)), \alpha \geq 1 \quad (15)$$

where  $u_1, \dots, u_{H_c}$  are the recalculated control input,  $\lambda_{\min}(\mathbf{A})$  the minimum eigenvalue of matrix  $\mathbf{A}$ ,  $\alpha$  the duality parameter. The magnitude of excitation for the identification can be tuned by varying the parameter  $\alpha$ . In the equation (12), the term

$$\sum_{k=0}^{H_c} \mathbf{Z}(k)\mathbf{Z}^T(k) \quad (16)$$

represents the increment of the information matrix in  $H_c$  steps. The control input satisfies the PE condition if the minimum eigenvalue of the information matrix is positive semi-definite:

$$\sum_{k=0}^{H_c} \mathbf{Z}(k)\mathbf{Z}^T(k) \geq \gamma \mathbf{I} \quad (17)$$

where  $\gamma$  is a scalar specifying the level of excitation and  $\mathbf{I}$  is a unit matrix of the corresponding dimensions.

The algorithm of maximization calculates control input vector  $\mathbf{U}(k)$ . The first input  $u_1$  is only applied to the control object.

### 2.3 Electromagnetic suspension model

Figure 1 shows a one-degree of freedom EMS model for our study. The control input is given as the applied voltage for the electromagnet, and we assume that the coil current is the measured output in the control system. In the modeling, the fringing effect and the leakage flux are neglected and the permeability of the magnetic material is constant for simplicity. The dynamics of the system is given by

$$M \frac{d^2x}{dt^2} + F - Mg = 0 \quad (18)$$

$$v = Ri + \frac{d}{dt}L(x)i \quad (19)$$

$$F = \frac{\mu_0 SN^2 i^2}{4 \left(x + \frac{l}{2\mu_r}\right)^2} \quad (20)$$

where  $x$  is the gap,  $M$  the mass,  $g$  the gravity acceleration,  $v$  the applied voltage,  $i$  the coil current,  $R$  the resistance,  $L(x)$  the self-inductance,  $F$  the attraction force,  $N$  the number of coil turns.

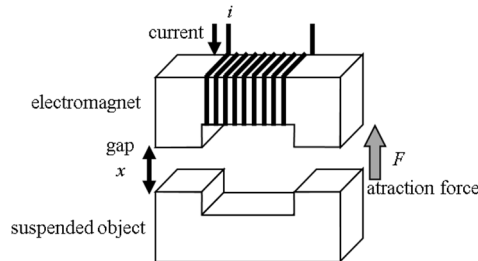


Fig.1 Electromagnetic suspension model

### 3. Simulation

Simulations of the approximate dual control for self-sensing electromagnetic suspension system are shown to see the effects of the duality parameter on the control performance. The ARX model for system is derived by using Eqs. (18) and (19). The initial parameter vector is  $\theta = (0.101 \ -0.210 \ 0.101 \ 2.970 \ -2.937 \ 0.969)^T$ . Table 1 summarizes the system parameters, while Table 2 shows the design parameters for the control system. The simulations (Lofberg, 2016) were carried out by setting the duality parameter as  $\alpha = 1, 2, \text{ and } 10$ . The reference gap is 1mm and the coil resistance was changed from  $0.3\Omega$  to  $0.322\Omega$  effect of parameter robustness. Figure 2 shows the responses of the control system for the period of  $0.05 - 0.4$  [s], which corresponds to the steady state of the numerical simulation. Figure 3 shows the estimated parameters, and Figure 4 shows the minimum eigenvalue of the information matrix.

In order to evaluate the cases for different values of the duality parameter, we calculated the root mean square error (RMSE), which is given by

$$x_{RMSE} = \sqrt{\frac{1}{N} \sum (r - x_i)^2} \quad (21)$$

$$\theta_{RMSE} = \sqrt{\frac{1}{N} \sum (\theta - \hat{\theta})^T (\theta - \hat{\theta})} \quad (22)$$

where  $N$  is the number of data. Table 3 shows the results for the dual control with different parameter settings. In Table 3,  $u_{var}$  denotes the variance of applied voltage and  $\bar{\lambda}$  denotes the average of minimum eigenvalue of the information matrix. When the duality parameter  $\alpha$  is large, it can be observed that the applied voltage fluctuations and the average minimum eigenvalue are large. Since the system obtained sufficient information to identify, the identification performance improved. However, too much excitation may cause unfavorable influence on the control performance so that we need to select a suitable duality parameter.

Table 1 System parameters

Name	Value
Permeability of free space	$4\pi \times 10^{-7}$
Mass of the floating object [kg]	1.06
Nominal relative permeability of iron	5000
The average length of the flux path [m]	0.27
Nominal air gap length [m]	$10^{-3}$
Nominal current [A]	1.027
Coil turns	280
Initial coil resistance [ $\Omega$ ]	0.3
Pole face area [m <sup>2</sup> ]	$4 \times 10^{-4}$
Sampling time [s]	0.002

Table 2 Controller parameters

Name	Value
$\sigma_z^2$	$10^{-5}$
$Q$	$10^{-8}I$
$Q_{MPC}$	1
$R_{MPC}$	1
$u_{max}$	5
$u_{min}$	-5

Table 3 the results for dual control with different parameter setting

	$x_{RMSE}$	$\theta_{RMSE}$	$u_{var}$	$\bar{\lambda}$
$\alpha = 1$	$1.76 \times 10^{-4}$	$1.02 \times 10^{-2}$	0.893	$9.89 \times 10^{-6}$
$\alpha = 2$	$1.40 \times 10^{-4}$	$8.20 \times 10^{-3}$	1.18	$5.58 \times 10^{-5}$
$\alpha = 10$	$3.81 \times 10^{-4}$	$8.17 \times 10^{-3}$	1.02	$7.33 \times 10^{-5}$

#### 4. Conclusion

This paper discussed the performance weight between the control and the identification by varying the design parameter for the self-sensing electromagnetic suspension with an approximate dual control technique. The simulation results show the magnitude of the duality parameter determines the weight for control and identification performances. High excitation improves the identification, while the control performance becomes worse. The performance weight is a tradeoff between the performances so that we have to decide an appropriate value depending on the purpose.

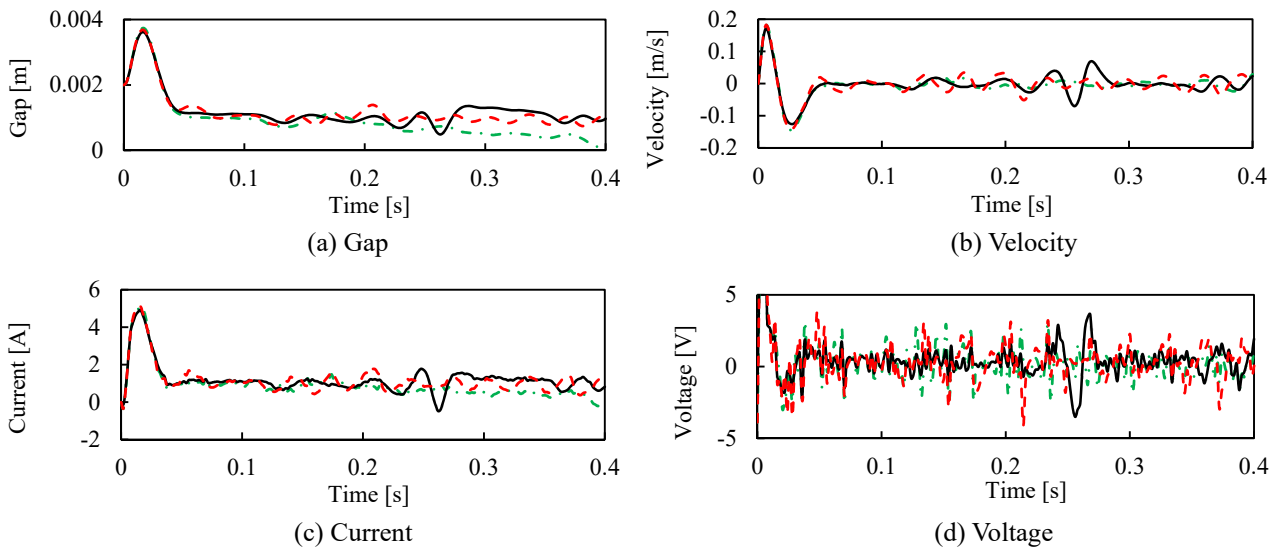


Fig. 2 The responses of the control system. The simulation of parameter  $\alpha = 1$  is plotted with the solid (black), parameter  $\alpha = 2$  is plotted with the dashed (red), parameter  $\alpha = 10$  is plotted with dashed and dotted (green), respectively.

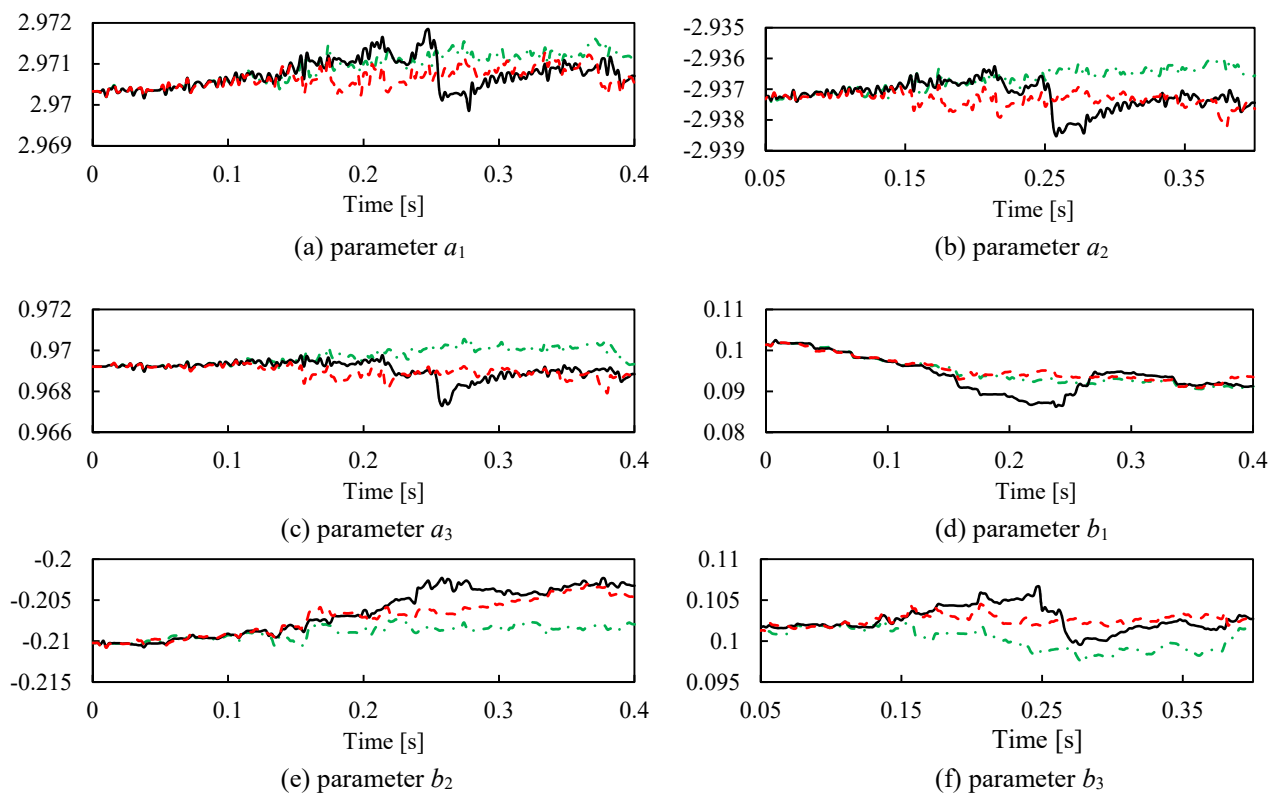


Fig. 3 The estimated parameters of ARX model using a Kalman filter. Parameter  $\alpha = 1$  is plotted with the solid (black), parameter  $\alpha = 2$  is plotted with the dashed (red), parameter  $\alpha = 10$  is plotted with dashed and dotted (green), respectively.

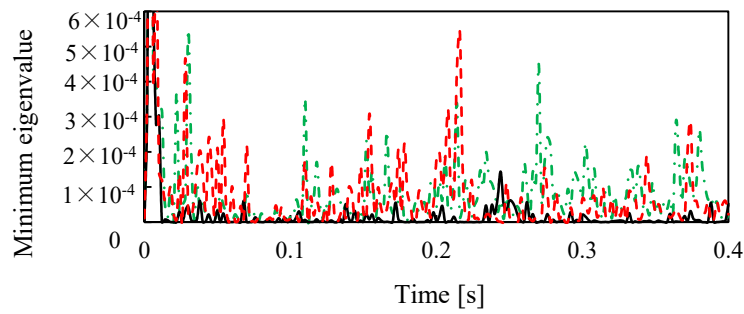


Fig. 4 Minimum eigenvalue of information matrix. Parameter  $\alpha = 1$  is plotted with the solid (black), parameter  $\alpha = 2$  is plotted with the dashed (red), parameter  $\alpha = 10$  is plotted with dashed and dotted (green), respectively.

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