THE EFFECT OF BODY'S AUTOROTATION IN ACTIVE MAGNETIC BEARINGS*)

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ABSTRACT

Main emphasis in this report is given to specific peculiarities of active magnetic bearings; these peculiarities lie in the capability of such bearings to ensure rotation of a ferromagnetic body magnetically suspended without use of a special motor. This capability is based upon the autorotation effect yielded by torque interaction of a magnetically-suspended ferromagnetic body with a suspension field when average per revolution (APR) torque affecting the body differs from zero.

The yielding conditions and peculiarities of the autorotation effect are exhibited in the example with a ferromagnetic body, of the shape very much similar to a magnetically-suspended ball.

INTRODUCTION

The effect of rotor autorotation in contact-free suspension has been predicted and revealed for the magnetic suspension by G.G. Denisov, Yu.I. Neimark and O.D. Pozdeev, who made a report on this subject at the Section of Navigational Systems at the Presidium of the USSR Academy of Sciences in 1962. By now, such rotor autorotation effect was spotted in superconductive [2] and electrostatic suspensions. In the latter, this effect was used to support stationary rotating speed of the rotor in a free gyroscope in the MICRON navigational system [1].

In precise gyroscopes, a torque rotor-suspension interaction results in a departure of direction of rotor kinetic momentum [3], and this is undesirable. When admissible, use of autorotation effect makes it possible to combine two functions in the contact-free suspension, viz. to be a support and a rotation motor. Here, the contact-free suspension actively affects a rotor in such a way that the centre of rotor's masses becomes stabilized in some specified position, and the angular speed of rotation increases up to some stationary value.

STRUCTURE OF A SIMPLEST MAGNETIC BEARING WITH THE AUTOROTATION EFFECT

The essence of autorotation effect is demonstrated (Fig. 1) on the simplest example of an active magnetic bearing (AMB) that holds a single electromagnet and uses as a rotor a ferromagnetic body with its shape very close to a ball. In Fig. 1 1 is an isotropic ferromagnetic rotor of the shape made slightly different from a ball by removing a cylinder face with its axis deviated from the vertical axis z of electromagnet 2 at angle φ; 3 is a sensor of gap δ between the rotor and the electromagnet core; 4 is a sensor of current I in the electromagnet; 5 and 6 are a proportional and a differentiating sections of the regulator in the AMB stabilizer; 7 is a power amplifier whose input receives a total regulating signal σ and whose output regulating voltage u is applied to the electromagnet.

MATHEMATICAL SIMULATION OF ACTIVE MAGNETIC BEARINGS WITH ROTOR'S TRANSLATIONAL AND ROTATIONAL DEGREES OF FREEDOM

To observe rotor's motions along two degrees of freedom (the translational, z, and rotational, φ) we introduce the Lagrange function

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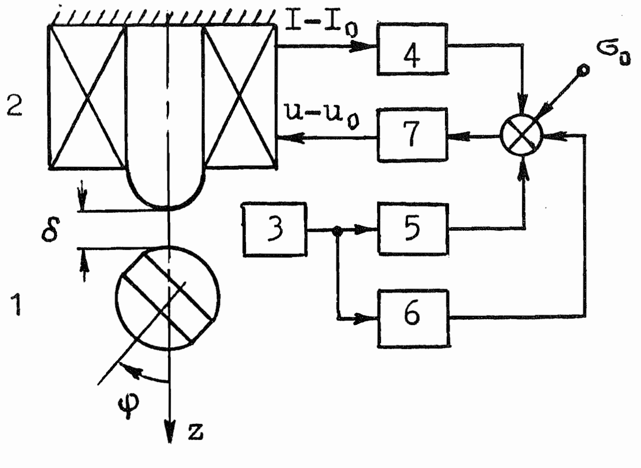

FIGURE 1

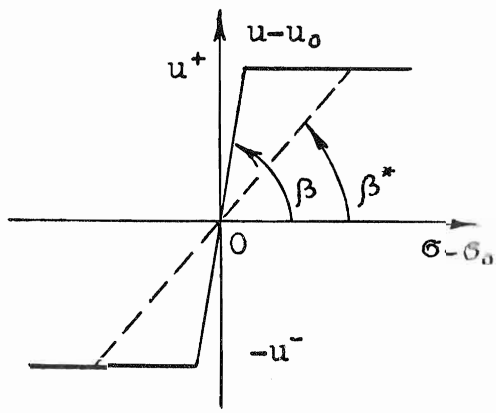

FIGURE 2

$$\mathcal{L} = m\dot{z}^2/2 + J\dot{\varphi}^2/2 - mgz + W(I,\delta), \quad (1)$$

where m and J stand for rotor's mass and inertia moment; g is acceleration of gravity, W is magnetic energy depending on current I in the electromagnet and gap δ between its core and the rotor. If the gap and the current deviate from their equilibrium values δ_0 and I_0 (that will correspond to a contact-free equilibrium state), magnetic energy may be represented by the series

$$W = W(I_0,\delta_0) + W_1(\delta-\delta_0) + W_2(I-I_0) + W_3(\delta-\delta_0)^2/2 + \\ + W_4(\delta-\delta_0)(I-I_0)/2 + W_5(I-I_0)^2/2 + \ldots, \quad (2)$$

where the coefficients reflect the following characteristics and parameters of the electromagnet:

$W_1 = \partial W/\partial\delta = F(I_0,\delta_0)$, an elevating force;
$W_2 = \partial W/\partial I = \Phi(I_0,\delta_0)$, a magnetic flux;
$W_3 = \partial W^2/\partial^2\delta = \partial F/\partial\delta = a$, a negative stiffness;
$W_4 = \partial W^2/\partial\delta\partial I = \partial F/\partial I = \frac{\partial\Phi}{\partial\delta} = b$; and
$W_5 = \partial W^2/\partial^2 I = L$ an inductance.

When values of the gap and the current in the electromagnet vary slightly, an elevating force of the electromagnet may be described in the linearized form

$$F = F(I_0,\delta_0) - a(\delta-\delta_0) + b(I - I_0) \quad (3)$$

where $F(I_0, \delta_0) = mg$, and a and b are constant parameters.
Dependence of gap deviations from the translational (z) and rotor's angular (φ) displacements is determined (Fig. 1) as

$$\delta - \delta_0 = z + \sum_k \rho_k \cos k\varphi \quad (4)$$

where ρ_k stands for the coefficients describing rotor surface nonsphericity.
Under these assumptions, rotor dynamics within a narrow neighborhood of stabilizable equilibrium state I_0, δ_0 will be described as

$$\begin{aligned} m\ddot{z} &= a(\delta-\delta_0) - b(I - I_0) \\ J\ddot{\varphi} &= M - F(I,\varphi)(\sum_k \rho_k \sin k\varphi)\cdot \\ L\dot{I} &= b\dot{z} - R\ I - I_0 + (u-u_0) \end{aligned} \quad (5)$$

Here there are taken into account the following dissipative characteristics of active magnetic bearings: M, a torque of friction; R, an active resistance of electromagnetic circuit. u, a voltage on the electromagnet, its equilibrium value being $u_0 = RI_0$.
In the stabilizing system (Fig. 1), the regulating algorithm suggested in [5] is used with due account of the following simplest piecewise-linear characteristic of the power amplifier:

$$u - u_0 = \begin{cases} u^+, & \beta(\sigma-\sigma_0) \ge u^+, \\ \beta(\sigma-\sigma_0), & -u^-\beta(\sigma-\sigma_0) \le u^+, \\ u^- & -u^- \ge \beta(\sigma-\sigma_0), \beta > \beta^*, \end{cases} \quad (6)$$

In Fig. 2 this characteristic is shown by solid lines where $u^{\pm}$ are levels of constraints for output voltage, and β is a linear amplification coefficient; here its value β^* is associated with the boundary of the stability region for the closed system (the Hurwitz angle denoted by a dash-dotted line).
The input of power amplifier receives the control signal

$$\sigma - \sigma_0 = \alpha\frac{\tau p + 1}{\varepsilon\tau p + 1}(\delta-\delta_0) - r(I - I_0) \quad (7)$$

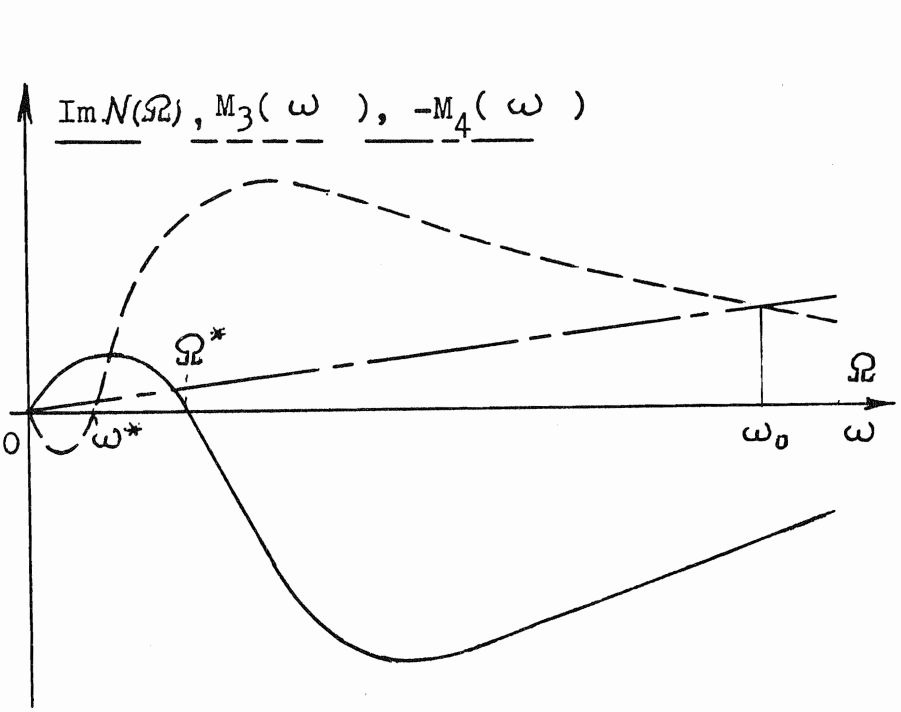

FIGURE 3

from gap sensors and current sensors, all of them possessing sensitivity α and r. A gap sensor signal is corrected by a differentiating section with use of time constant τ and error $\varepsilon < 1$. Here p stands for a differentiating operator.

The control algorithm (6–7) is widely applied to active magnetic bearings and, as shown in [5] when choosing optimal values of control parameters, it ensures a maximum region of attraction of stabilizable equilibrium state in the state space of the system.

THE EFFECT OF AUTOROTATION IN THE ACTIVE MAGNETIC SUSPENSION

Dynamics of rotation of a nonspheric rotor in the active magnetic suspension is mathematically simulated in Eqs. (3)-(7) under the assumption that the rotor rotation angle φ varies under the equation

$$\varphi = \varphi_0 + \omega t \,, \qquad (8)$$

where φ_0 is an initial value of this angle, and ω is a sufficiently high speed of rotor rotation. Such assumption allows us to study Eqs. (3)-(8) by averaging (Mitropolsky [4]) with respect to explicitly incoming times.

In this case, the equilibrium state of the system averaged on axis z will be described as

$$\langle z \rangle = -\delta_0 , \qquad (9)$$

i.e., in this state the centre of rotor masses becomes stabilized, and as a result of the rotor shape deviating from the spherical one the signal of the displacement sensor will be modulated synchronously to rotor rotation.

Now let us study the torque acting upon steadily suspended rotating nonspheric rotor from the side of the active magnetic bearings. For this, linearized power characteristics (3) will be introduced into the second equation of system (5) and we shall calculate the APR torque representing it as a sum of four constituents

$$2\pi J\dot{\omega} = \sum_{j=1}^{4} M_j \,, \qquad (10)$$

where

$$M_1 = -F(\delta_0, I_0)\left(\sum_k k\rho_k \int_0^{2\pi} \sin k\varphi d\varphi\right) = 0$$

$$M_2 = a\left(\sum_k k\rho_k^2 \int_0^{2\pi} \cos k\varphi \sin k\varphi d\varphi\right) = 0$$

$$M_3 = -b\left(\sum_k k\rho_k \int_0^{2\pi} (I - I_0) \sin k\varphi d\varphi\right)$$

$$M_4 = \int_0^{2\pi} M d\varphi$$

The first two constituents of the torque are of a conservative nature, i.e., in average one revolution of the rotor brings $M_1 = M_2 = 0$. The third constituent of the torque depends on the phase ratio between a variable constituent of electromagnet current I and an angular turn φ of the rotor (a constant constituent of the current I_0 does not contribute to an APR torque; it is analogous to the above case). To calculate M_3, the variable constituent of the electromagnet current is easier to represent as two terms:

$$I = \left(\sum_k I_{sk} \cos k\varphi\right) - \left(\sum_k I_{sk} \sin k\varphi\right) .(11)$$

Here, the first term coincides in phase with departures of the gap (4) during rotor rotation; and the second term lags behind at $\pi/2$ (or goes ahead if the sign of amplitudes I_{sk} of harmonics turns to the opposite). Substituting Eq. (11) into M_3 reveals the second term to determine nonzero APR torque affecting the rotor from the side of the active magnetic bearing.

To calculate it, the variable constituent of the current associated with the second term in Eq. (11) is determined with the help of imaginary portion of the gap-current transfer function ($N(\Omega)$-function) of the stabilizing system in the active magnetic bearing. In Fig. 3 the dependence of this constituent upon frequency Ω of the harmonic signal is designated by a solid line. With regard of the fact that there exists a relation between frequency Ω of the nonsphericity harmonic signal and rotor rotation frequency ω, we shall obtain

$$M_3 = -b\left(\sum_k k\rho_k \operatorname{Im} N(k\omega)\right)\bigg/2, \quad k\omega = \Omega \, .(12)$$

For the active magnetic bearing under study, in the rotor, when well-balanced, the main contribution to this constituent of the torque is brought by the harmonic with minimal value, $k = 2$. The qualitative shape of $M_3(\omega)$ is shown in Fig. 3 by a dotted line. On this basis it may be asserted that the above active magnetic bearing will damp rotation of the nonspheric rotor within the rotation frequency interval $[0–\omega^*]$ and, for $\omega > \omega^*$, speed up the rotor until some speed ω_0 specified by $M_3 = M_4$. Here M_4 stands for the dissipative constituent of the APR torque; when there is no Coulomb friction, this constituent is supposed to linearly depend upon the rotor rotation speed (shown by the dash-dotted line in Fig. 3).

Decreasing M_4 makes it possible to speed up the rotor with torque M_3 up to the speeds that are practically constrained only by strength of its material. This has been proved experimentally through placing the rotor into a vacuum chamber.

CONCLUSION

Thus, we have shown a possibility of the rotor to steadily rotate in active magnetic bearings without special motor used for support in a stationary speed of this rotation, which is the essence of the effect of autorotation.

The predicted and implemented effect of autorotation makes it possible to simplify a design of rotor systems and it is already applied to a number of gyroscopic devices with contact-free rotor suspension.

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