8th International Symposium on Magnetic Bearing
August 26-28, 2002, Mito, Japan

INFLUENCES OF THRUST MAGNETIC BEARING AND JOURNAL TILT ON DYNAMIC CHARACTERISTICS OF ROTOR SYSTEM

Gang Zhang
Research Institute of Bearing, Shanghai University, Shanghai, 200072, China
gzhang@mail.shu.edu.cn, gzhanghd@sh163.net
Jiansheng Zhang
Research Institute of Bearing, Shanghai University, Shanghai, 200072, China
g.h.cnc@public.nt.js.cn
Xiping Wang
Research Institute of Bearing, Shanghai University, Shanghai, 200072, China
wxp_sh_fl@citiz.net
Lie Yu
Theory of Lubrication and Bearing Institute, Xi'an Jiaotong University, Xi'an, 710049, China,
tlbi@sun20.xjtu.edu.cn
Youbai Xie
Theory of Lubrication and Bearing Institute, Xi'an Jiaotong University, Xi'an, 710049, China,
ybxie@mail.xjtu.edu.cn

ABSTRACT
The influences of thrust magnetic bearing and journal tilt on dynamic characteristics of rotor system are studied mainly in this paper. The outcome shows: the coupling effect of thrust magnetic bearing has principally influence on low-frequency taper vortex, the damping inherent frequency of relevant eigenvalue is increased along with the decrease of dimensionless damping, the system stability of relevant vibration mode reduces. The influences of coupling effects of thrust magnetic bearing and tilt of journal on critical rotational speeds of system are very large, and the stability of system reduces greatly. Under some set of control parameters near the border in stable region, in the ideal condition of rotor in exact alignment of magnetic bearings, the system is stable when the coupling effect of thrust magnetic bearing is neglected in the calculating model, and the system is instable when the coupling effect of thrust magnetic bearing is considered in the model. In the general condition of rotor tilting in the magnetic bearings, the system is sable when the coupling effects of thrust magnetic bearing and radial magnetic bearings due to rotor tilting are neglected in the calculating model, and the system is instable when the coupling effects is considered in the model. The stable system in the ideal condition of rotor in exact alignment of magnetic bearings may become instable in the general condition when the rotor tilts in the magnetic bearings. That shows it is important that the influence of coupling effects of thrust and radial magnetic bearings due to the rotor tilting must be considered in the study of rotor-magnetic bearings system dynamic characteristics, and also shows it is important that the rotor is mounted and debugged in ideal alignment of magnetic bearings.

INTRODUCTION

Since 1990, an important discovery in rotor-hydrody -namic bearing system dynamics is the coupled action of thrust bearing upon the lateral vibration in a rotor system. In the pioneering research of Mittwollen *et al.* (1991) [1], a series of rotor-dynamic coefficients of thrust hydrodynamic bearing were defined and used for the investigation of the action of thrust hydrodynamic bearing upon the lateral vibration in which the static tilt of the runner was neglected. Afterwards, in the research of Lie Yu *et al.* (1995) [2], a general analysis method was developed to investigate the coupled dynamics of a rotor equipped with journal and thrust hydrodynamic bearings simultaneously. Considerations included the effects of static tilt parameters of the rotor on rotor-dynamic coefficients of thrust hydrodynamic bearing and the action of thrust hydrodynamic bearing upon the system dynamics. Prof. Zhiming Zhang (1963) [3] solved the Reynolds Equation for oil pressure distribution in journal sliding bearing with the oil pressure as a power series of journal eccentricity and tilting ratio and determined the trajectory of the journal center and bearing capacity. However, the coupled actions of thrust magnetic bearing and journal tilt in radial magnetic bearings upon the lateral vibration in a rotor system have not been studied thoroughly in a public literature.

When getting into 21 century, research of Gang Zhang *et al.*(2000) [4, 5] studied the static and dynamic mechanical characteristics of a thrust and radial magnetic bearing due to the rotor inclination. The calculation formulas of static and dynamic mechanical characteristics of the thrust and radial magnetic bearings were derived. Application refers to a thrust magnetic bearing and two radial magnetic bearings for a turbo-expander/compressor. The static tilt of the rotor has remarkable influence on the mechanical character -istics of thrust and radial magnetic bearings, changes the static load distribution between two radial magnetic bearings and will exert violent coupling effect among a thrust magnetic bearing and two radial magnetic bearings. This paper will use such a finding for the coupled electromechanical dynamics analysis of rotor system equipped with thrust and radial magnetic bearings, and the influence of thrust magnetic bearing and journal tilt on dynamic characteristics of rotor system is studied mainly.

EQUATION OF STATE

Consulting the dissertation [6], the electromechanical coupled dynamic equations of a rotor-active magnetic bearings system are:

$$\begin{cases} M_B\omega^2 q_B'' + G_B\omega q_B' + K_B q_B + K_{IB} I_B = F \\ T_a\omega I_B'' + I_B' + A_a U_{out}' = 0 \\ T_s\omega U_e'' + U_e' + A_s L_{SB} q_B' = 0 \\ -T_{dr}\omega^2 U_{out}'' - \omega U_{out}' + (K_{pi} - K_{dp}T_s^{-1} - K_{ir}T_s)\omega U_e' \\ -K_{dp}T_s^{-1}A_s L_{SB}\omega q_B' - K_{ir}A_s L_{SB} q_B = 0 \end{cases} \tag{1}$$

When the input state variable $X_m = (q_B, I_B, q_B', I_B', U_e', U_{out}')^T$ and output state variables Y_m=q_B, U_m=$F(t)$ are supposed, the relevant equation of state is:

$$\begin{cases} X_m' = A_m X_m + B_m U_m \\ Y_m = C_m X_m + D_m U_m \end{cases} \tag{2}$$

with

$B_m = (0_{5\times5},\ 0_{5\times5},\ M_B^{-1}\omega^{-2},\ 0_{5\times5},\ 0_{5\times5},\ 0_{5\times5})^T$

$C_m = [I_{5\times5},\ 0_{5\times5},\ 0_{5\times5}, 0_{5\times5}\ ,\ 0_{5\times5},\ 0_{5\times5}]$,

$D_m = 0_{5\times5}$

$$A_m = \begin{bmatrix} 0_{5\times5} & 0_{5\times5} & I_{5\times5} & 0_{5\times5} & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & 0_{5\times5} & I_{5\times5} & 0_{5\times5} & 0_{5\times5} \\ -M_B^{-1}\omega^{-2}K_B & -M_B^{-1}\omega^{-2}K_{IB} & -M_B^{-1}\omega^{-1}G_B & 0_{5\times5} & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & 0_{5\times5} & -T_a^{-1}\omega^{-1} & 0_{5\times5} & -T_a^{-1}\omega^{-1}A_a \\ 0_{5\times5} & 0_{5\times5} & -T_s^{-1}\omega^{-1}A_s L_{SB} & 0_{5\times5} & -T_s^{-1}\omega^{-1} & 0_{5\times5} \\ -T_{dr}^{-1}\omega^{-2}K_{ir}A_s L_{SB} & 0_{5\times5} & -T_{dr}^{-1}\omega^{-1}K_{dp}T_s^{-1}A_s L_{SB} & 0_{5\times5} & T_{dr}^{-1}\omega^{-1}(K_{pi}-K_{dp}T_s^{-1}-K_{ir}T_s) & -T_{dr}^{-1}\omega^{-1} \end{bmatrix} \tag{3}$$

The electromechanical coupled dynamic characteristics such as complex eigenvalues, complex modes, critical rotational speeds, instable rotational speed, forced vibration response and so on can be calculated easily from equation (2) with Matlab language.

ANALYSIS Of CHARACTERISTICS

Fig.1 is the schematic of a rotor-magnetic bearing sys -tem in a turboexpander/compressor. A set of parameters in calculation are as follows: the thrust load W_{0t}=1500N, the thrust magnetic bearing clearance c_{0t}=0.5mm, the number of coil N_t=143, the bias current I_{0t}=4.0A, the distance between two radial magnetic bearings l=0.275 m, the weight of rotor W_{0r}=123N, the bore of radial magnetic bearings D=61.4mm, the radial clearance c_{0r}=0.4mm, the pole number N_p=8, the number of each pole N_r=57, the bias current I_{0r}=4.0A.

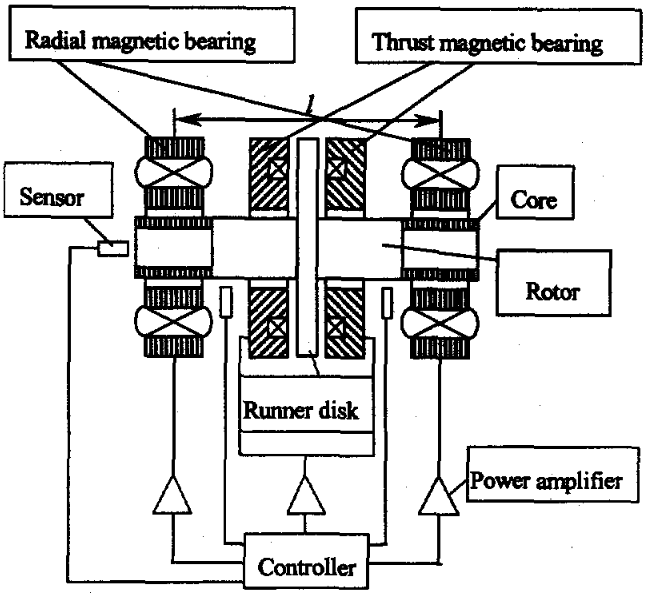

FIGURE 1: 5-axis controlled rotor-magnetic bearing system in a turbo-expander/compressor

Influence of Thrust Magnetic Bearing on Dynamic Characteristics of Rotor System

Taking the ideal condition of rotor in exact alignment of bearings, or the position of rotor in the bearings $x_{a0}=y_{a0}=x_{b0}=y_{b0}=z_{c0}=0$ for example, the parameters of controller are decided on the main diagonal elements in the optimum feedback matrix with LQ method. They are the proportional coefficients $k_{prxa}=k_{prya}=k_{prxb}=k_{pryb}=$ 3.8333, k_{przc}=3.625, the integral coefficients $k_{irxa}=k_{irya}=k_{irxb}=k_{iryb}=k_{irzc}$=197.0055$s^{-1}$, the differential coefficients $k_{drxa}=k_{drya}=k_{drxb}=k_{dryb}=k_{drzc}$=0.0092s, the differential time attenuation constants $T_{drxa}=T_{drya}=T_{drxb}=T_{dryb}=T_{drzc}=$ 6.2422×10^{-4}s, the power amplifier time attenuation constants $T_{axa}=T_{aya}=T_{axb}=T_{ayb}=T_{azc}=3.2\times10^{-5}$s, the sensor time attenuation constants $T_{sxa}=T_{sya}=T_{sxb}=T_{syb}=T_{szc}=3.2\times10^{-5}$s. The complex eigenvalue $\lambda_i=U_i+iV_i$ can be calculated when the coupling effect of thrust magnetic bearing is considered. Fig.2 is the complex modes of free vibration when the rotor runs in rotation speed n=40000rpm.

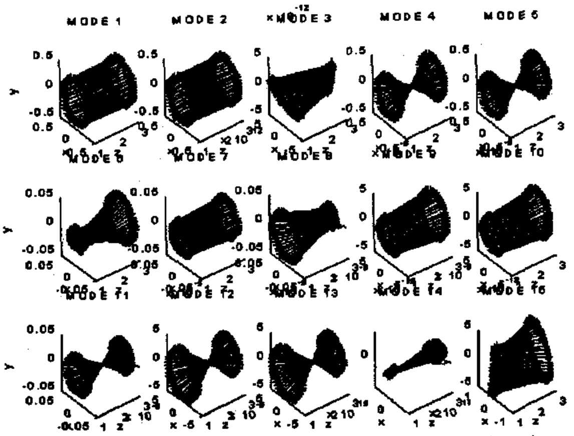

FIGURE 2: The complex modes of free vibration with n=40000rpm in ideal condition.

Fig.2 shows that there are 15 pairs of conjugate complex roots and 15 complex modes. They correspond three types of vibrations. Or they are free vibration of mechanical system, free vibration of electrical system and free vibration of electromechanical coupling system.

The low frequency vibration is mainly discussed here. The mode 1, mode 2 and mode 3 in the first five modes corresponds respectively columnar vortex and axial vibration of rotor. The thrust magnetic bearing does not exert an influence on the three modes. Or the influence is very small. Their physical meaning is clearly that the moment stiffness coefficients of thrust magnetic bearing relate only to the small perturbation of rotor angular. The negative moment stiffness depends only on the difference and photo-position of displace -ment perturbation. The influence of thrust magnetic bearing is obviously negligible under the two modes of columnar vortex and axial vibration.

We pay closest attention to the mode 4 and mode 5. They correspond the typical taper vortex. When the coupled effect of thrust magnetic bearing is neglected in the calculation, the two eigenvalues are respectively (-0.0293±0.0075i), (-0.0346±0.0106i) with n=40000rpm. When the coupled effect of thrust magnetic bearing is considered in the calculation, the two eigenvalues are evolved respectively (-0.0283±0.0142i), (-0.0305± 0.0172i) with n=40000rpm. The damping inherent frequencies are increased respectively by factors of 89.3% and 62.3%. The absolute values of real part in the eigenvalues are decreased respectively by factors of 3.4% and 11.8%. That is the dimensionless damping reduces, the stability of the modes in the system reduces.

Influence of Journal Tilt on Dynamic Characteristics of Rotor System

Taking the general condition of rotor tilting in the bearings, or the position of rotor in bearings $x_{a0}=y_{a0}=x_{b0}=y_{b0}=0.4$, $z_{c0}=0$ for example, the parameters of controller are maintained the same as preceding level. Owing to the large change of the dynamic coefficients in the thrust and radial magnetic bearings [5], the 12 stiffness coefficients of thrust magnetic bearing and 24 stiffness coefficients of radial magnetic bearings are not equal to zero. Comparing with the ideal condition of rotor in exact alignment of bearings, the great changes of the real part and imaginary part of every eigenvalue have taken place. They are changed into the mix vibration composed of taper and columnar vortexes as shown in fig.3.

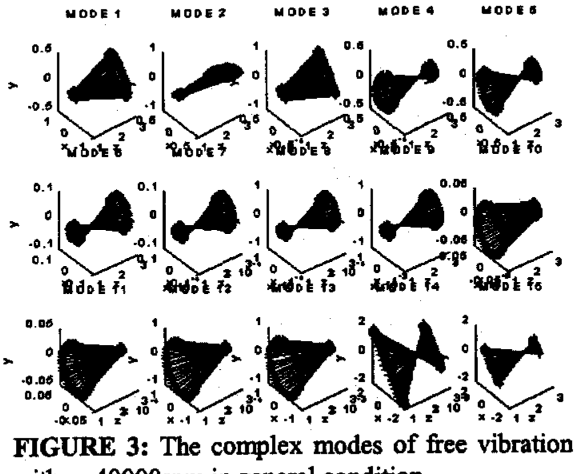

FIGURE 3: The complex modes of free vibration with n=40000rpm in general condition.

Taking the eigenvalues of n=40000rpm for example, great changes of the real and imaginary part of every eigenvalue have taken place. Comparing the calculating model in that the influence of journal tilt is neglected, 73.6% and 82.7% are respectively the largest relative change ratio of real part and imaginary part of the eigenvalues calculated according to the model that the influence of journal tilt is considered.

When the influences of thrust magnetic bearing and journal tilt are neglected the critical rotational speeds of the system are n_{ci}=19154rpm, 20190rpm, 22671rpm, 22692rpm, 24060rpm, 24064rpm, 26407rpm, 35191rpm, 37507rpm. When the coupling influences of thrust magnetic bearing and journal tilt are considered the critical rotational speeds of the system are evolved respectively as n_{ci}=20831rpm, 21936rpm, 24341rpm, 24358rpm, 30263rpm, 30667rpm, 34340rpm, 36785rpm, 39029rpm. Comparing the ideal condition that the coupling influence of thrust magnetic bearing and journal tilt are neglected, 23.5% is the largest relative change ratio. And the smallest absolute value 0.0293 of negative real part of eigenvalue in the ideal condition decreases to 0.0051 in general condition. The stability decreases by 82.6%.

The Coupling Influences of Thrust Magnetic Bearing and Journal Tilt on Dynamic Characteristics of Rotor System Under the Condition near the Border in the stable region

The Ideal Condition of Rotor in Exact Alignment of magnetic Bearings. Taking the ideal condition of $x_{a0}=y_{a0}=x_{b0}=y_{b0}=z_{c0}=0$ for example, the parameters of controller are set at $k_{prxa}=k_{prya}=k_{prxb}=k_{pryb}=k_{przc}=2.4231$, $k_{irxa}=k_{irya}=k_{irxb}=k_{iryb}=k_{irzc}=139.9776\text{s}^{-1}$, $k_{drxa}=k_{drya}=k_{drxb}=k_{dryb}=k_{drzc}=0.0037\text{s}$, $T_{drxa}=T_{drya}=T_{drxb}=T_{dryb}=T_{drzc}=2.5801\times10^{-4}\text{s}$, $T_{axa}=T_{aya}=T_{axb}=T_{ayb}=T_{azc}=3.2\times10^{-5}\text{s}$, $T_{sxa}=T_{sya}=T_{sxb}=T_{syb}=T_{szc}=3.2\times10^{-5}\text{s}$. The every eigenvalue $\lambda_i=U_i+iV_i$ is calculated respectively according to the models that the influence of thrust magnetic bearing is neglected and is considered. Then the every critical rotational speed and instable rotational speed are calculated with interpolation. Fig.4 and Fig.5 are respectively the root loci of the first 5 eigenvalues with n=5000~40000rpm and n=105000~140000rpm when the influence of thrust magnetic bearing is neglected. Fig.6 is the root loci of the first 5 eigenvalues with n=5000~40000rpm when the influence of thrust magnetic bearing is considered.

It may be seen that under this set of control parameters, the instable rotational speed is n_{sw}= 127500rpm when the influence of thrust magnetic bearing is neglected. The system is stable within the range of operating rotational speed n=40000rpm. However, the instable rotational speed is n_{sw}=32778rpm when the influence of thrust magnetic bearing is considered. The system is instable within the range of operating rotational speed n=40000rpm. That shows that it is important that the influence of coupling effect of thrust magnetic bearing must be considered in the study

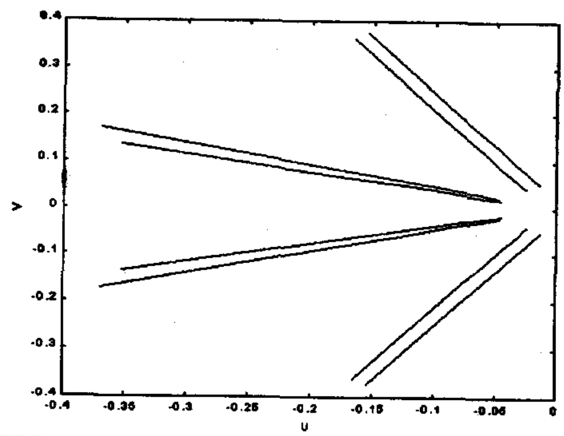

FIGURE 4: The root loci of the first 5 eigenvalues with n=5000~40000rpm when the influence of thrust magnetic bearing is neglected.

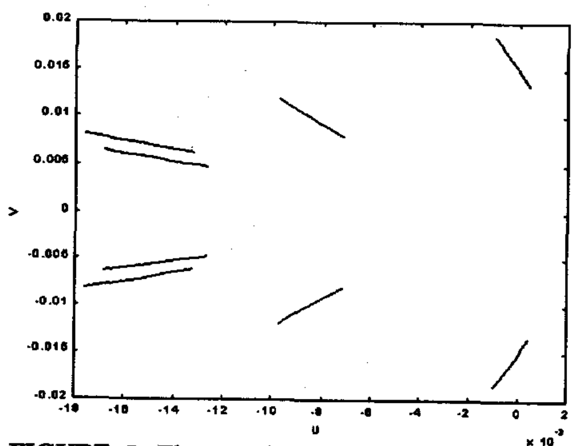

FIGURE 5: The root loci of the first 5 eigenvalues with n=105000~140000rpm when the influence of thrust magnetic bearing is neglected.

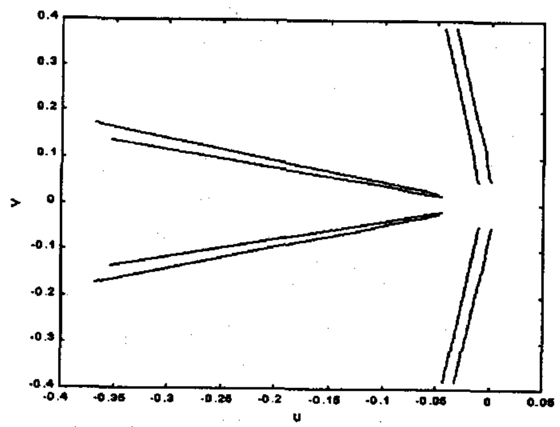

FIGURE 6: The root loci of the first 5 eigenvalues with n=5000~40000rpm when the influence of thrust magnetic bearing is considered.

of rotor-magnetic bearings system dynamic character -istics.

The General Condition of Rotor tilting in the Bearings. Taking the general condition of $x_{a0}=y_{a0}=-0.4$, $x_{b0}=y_{b0}=0.4$, $z_{c0}=0$ for example, the control parameters $k_{prxa}=k_{prya}=k_{prxb}=k_{pryb}=k_{przc}=3.4412$, $k_{irxa}=k_{irya}=k_{irxb}=k_{iryb}=k_{irzc}=183.4189s^{-1}$, $k_{drxa}=k_{drya}=k_{drxb}=k_{dryb}=k_{drzc}=0.0082s$, $T_{drxa}=T_{drya}=T_{drxb}=T_{dryb}=T_{drzc}=5.5951\times10^{-4}$s, $T_{axa}=T_{aya}=T_{axb}=T_{ayb}=T_{azc}=3.2\times10^{-5}$s, $T_{sxa}=T_{sya}=T_{sxb}=T_{syb}=T_{szc}=3.2\times10^{-5}$s, the every eigenvalue $\lambda_i=U_i+iV_i$ is calculated respectively according to the models that the influences of thrust magnetic bearing and journal tilt are neglected and are considered. Then the every critical rotational speed and instable rotational speed are calculated with interpolation. Fig.7 and Fig.8 are respectively the root loci of the first 5 eigenvalues with n=5000~40000rpm when the influences of thrust magnetic bearing and journal tilt are neglected and are considered. Fig.9 is the root loci of the first 5 eigenvalues with n=5000~40000rpm when the influences of thrust magnetic bearing and journal tilt are considered and the rotor turns into exact alignment of bearings.

It may be seen that under this set of control parameters, within the range of operating rotational speed n=40000rpm the system is stable (the instable rotational speed n_{sw}>70000rpm) when the influences of thrust magnetic bearing and journal tilt are neglected and the system is instable (the instable rotational speed n_{sw}=17500rpm) when the influences of thrust magnetic bearing and journal tilt are considered. However, the system becomes stable again (the instable rotational speed n_{sw}>72407rpm) when the influences of thrust magnetic bearing and journal tilt are considered and the rotor turns into exact alignment of bearings. That shows again that it is important that the influences of coupling effects of thrust magnetic bearing and journal tilt must be considered in the study of rotor-magnetic bearings system dynamic characteristics and also shows that it is important that the rotor is mounted and debugged in ideal alignment of magnetic bearings.

These characteristics shown above have been verified by experiments of late years.

CONCLUSIONS

The outcome shows:

(1) The coupling effect of thrust magnetic bearing has principally influence on low-frequency taper vortex, the damping inherent frequency of relevant eigenvalue is increased along with the decrease of dimensionless damping, the system stability of relevant vibration mode reduces.

(2) The influences of coupling effects of thrust magnetic bearing and tilt of journal on critical rotational

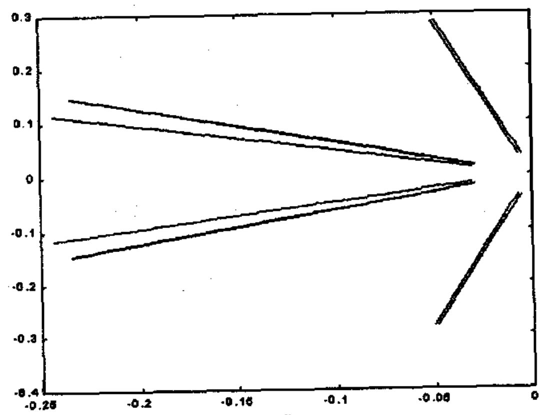

FIGURE 7: The root loci of the first 5 eigenvalues with n=5000~40000rpm when the influences of thrust magnetic bearing and journal tilt are neglected.

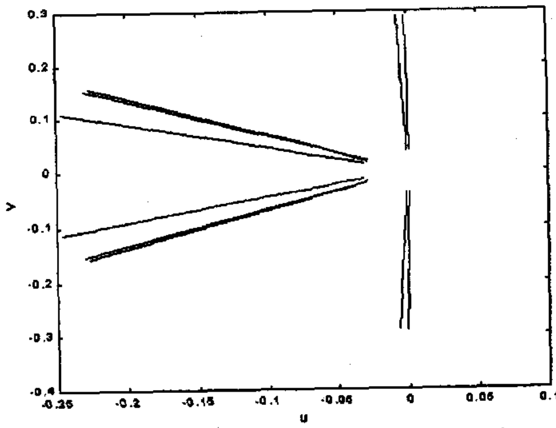

FIGURE 8: The root loci of the first 5 eigenvalues with n=5000~40000rpm when the influences of thrust magnetic bearing and journal tilt are considered.

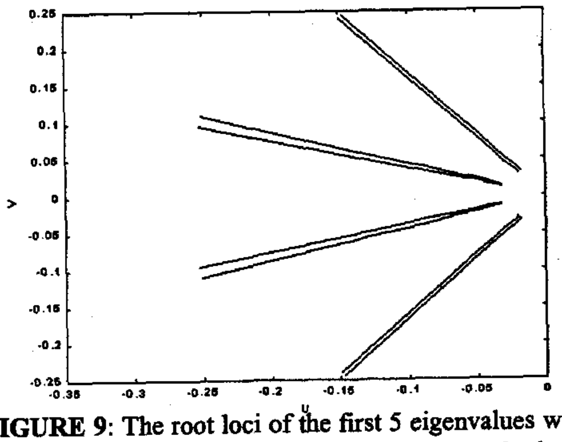

FIGURE 9: The root loci of the first 5 eigenvalues with n=5000~40000rpm when the influences of thrust magnetic bearing and journal tilt are considered and the rotor turns into exact alignment of bearings.

speeds of system are very large, and the stability of system reduces greatly.

(3) Under some set of control parameters near border in the stable region, in the ideal condition of rotor in exact alignment of bearings, the system is stable when the coupling effect of thrust magnetic bearing is neglected in the calculating model, and the system is instable when the coupling effect of thrust magnetic bearing is considered in the model. In the general condition of rotor tilting in the bearings the system is stable when the coupling effects of thrust magnetic bearing and radial magnetic bearings due to rotor tilting are neglected in the calculating model, and the system is instable when the coupling effects is considered in the model. The stable system in the ideal condition of rotor in exact alignment of bearings may become instable in the general condition when the rotor tilts in the bearings. That shows it is important that the influences of coupling effects of thrust and radial magnetic bearings due to the rotor tilting must be considered in the study of rotor-magnetic bearings system dynamic characteristics, and also shows it is important that the rotor is mounted and debugged in ideal alignment of magnetic bearings.

ACKNOWLEDGEMENTS

The thesis was sponsored by the Special Research Fund for Doctorate Subjects in Universities of the National Educational Committee: Coupled Electromechanical Dynamics of a Rotor-Active Magnetic Bearings System (No.9469834). The research on which this paper is based has been supported by the National 863 Project Science Foundation of Chinese Science and Technology Ministry: Supporting Technology of Magnetic Suspensions in Senior NC Machine Tools (Cont. No. 2001AA423310).

REFERENCES

1 N.Mittwollen, T.Hegel, J.Glienicke. Effect of Hydro -dynamic Thrust Bearings on Lateral Shaft Vibrations. Journal of Tribology, Trans. ASME, 1991, 113: 811~818

2 Lie Yu, R.B.Bhat. Coupled Dynamics of a Rotor -Journal Bearing System Equipped with Thrust Bearings. Shock and Vibration, 1995,2(1): 1~14

3 Zhiming Zhang , A theoretical study of the oil pressure distribution and load-carrying Capacity of 360° sliding bearings with inclined journals, JOURNAL OF XI'AN JIAOTONG UNIVERSITY, 1963, (1): 50～61

4 Gang Zhang, Xiping Wang, Lie Yu, Youbai Xie. Characteristics of Thrust Magnetic Bearing and its Effect on Radial Ones in the System. Proc. of 7th Int. Symp. on Magnetic Bearings, ETH Zurich, Switzerland, August, 2000: 579～582

5 Gang Zhang, Lie Yu, Youbai Xie. Mechanical Characteristics of a Radial Magnetic Bearing. Proc. of 1st Int. Conference on Mechanical Engineering, ICME 2000: 61～62

6 Gang Zhang. Coupled Electromechanical Dynamics of a Rotor-Active Magnetic Bearings System. Dissertation Submitted to Xi'an Jiaotong University for the Degree of Doctor of Engineering, Xi'an, China, 1999